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Condition for Zero Doppler Effect in Classical Theory, Special Relativity and New Approach

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ABSTRACT: *The Doppler effect (DE) has a wide application in many areas of science and technology. Radiofrequency technologies that use this effect are based on the classical DE theory. We know that since 1905, we have the relativistic theory of DE. Recently, a new DE theory based on relative velocity between wavefronts and wave source/observer has been proposed. In this paper, we will examine the condition for the vanishing of DE, that is, no frequency shift irrespective of the relative motion between the wave and the wave source/observer. Examination will be done to the three theories of DE.*

Keywords: Doppler effect, zero Doppler effect, relative velocity, relativity, electromagnetism.

1. INTRODUCTION

In 1842, C. Doppler has shown that if a moving star emits light with the wavelength, then an observer at rest will observe this light with the wavelength , which can be expressed by the following equation:

$$\lambda_o = \lambda_s \frac{c \pm v}{c}, \tag{1}$$

where is the velocity of light in free space and is the speed of the moving star.[1] If the direction of the observer's sight to the source makes an angle ϑ, the following formula was proposed:

$$\lambda_o = \lambda_s \frac{c + v\cos\vartheta}{c}. \tag{2}$$

This formula is known as the classical Doppler effect (DE) formula. Doppler's theory was strongly criticised around 1852 by J. Petzval.[2] Petzval argued that no great science could come from a few simple lines of algebra. In Petzval's view, all phenomena in nature were the manifestations of underlying differential equations. In order to explain the null result of Michelson experiment, W. Voigt proposed a new analysis of the Doppler effect by using the wave equation (which is a partial differential equation).[3] In his paper, Voigt obtained a space time transformation that is often considered as the origin of Lorentz Transformation and thus, of special relativity. In 1905, in his special theory of relativity, Einstein introduced a new formula for the Doppler effect for any angle ϑ, which can be written for a moving source:

$$\lambda_o = \lambda_s \frac{c + v\cos\vartheta}{c\sqrt{1 - \frac{v^2}{c^2}}}. \tag{3}$$

The only difference with the classical DE formula is the division by the Lorentz factor, which is justified by the theory of relativity with time dilation. In problems related to particle accelerators, it has been shown that the relativistic DE formula gives better prediction than the classical DE formula.[4–6] These experiments are often used as experimental evidence of the special theory of relativity.

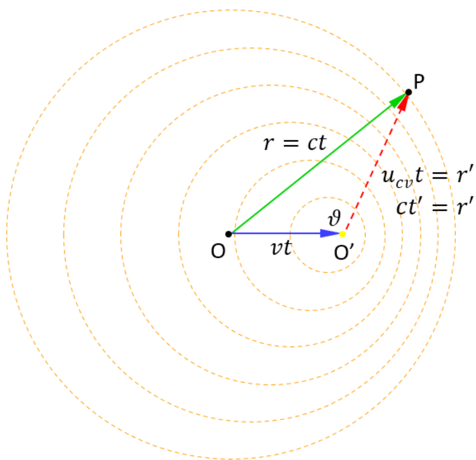

Figure 1: Diagram for analysing the Doppler effect – the source moves and the observer P is at rest.

In 1986, J. P. Wesley wrote the following about Voigt's analysis of the Doppler effect and its consequences: "Voigt thereby obtained the relations that are today inappropriately called the Lorentz transformation. Voigt represented his Doppler effect mathematically in terms of space and time variables, whereas the Doppler effect can involve the propagation constant and frequency only. Voigt's unfortunate mathematical representation of his Doppler effect in terms of space and time apparently led Lorentz and others to naively conclude that space and time themselves might actually change in a moving system".[7] The author, in 2018 also reviewed Voigt's paper.[8]

A new approach was recently proposed for calculating the general formula of the DE.[9–13] This approach consists in deriving the parameter has shown in Figure 1. This parameter represents the relative velocity between wavefronts coming from the source and the observer. In Figure 1, we see dashed circles that represent wavefronts emitted by the source, which moves at velocity . At point P, the observer rests and receives these wavefronts with relative velocity u_{cv}, which multiplied by t gives ŕ. Distance ŕ, which is the distance between the source and the observer can also be expressed by product ct' , where is the velocity of wavefronts and t' is the time, within which the observer receives the wavefronts emitted by the source within time . By using this method, and the law of cosines it can be shown that the observer will receive the following wavelength:

$$\lambda_o = \lambda_s\left(\sqrt{1-\frac{v^2}{c^2}sin^2\vartheta}+\frac{v}{c}cos\vartheta\right). \quad (4)$$

It can be shown that this formula also predicts a transverse DE, so the experiments that support the relativistic DE formula (3) with respect to the transverse DE (as a special case of DE), also should be compared to the new DE formula (4).[4–6] On the other hand, the alternative theory does not use time dilation, but only a classical approach based on ray technique and geometry, to derive the DE. This formula by W. Y. Tey is compared with the results of a numerical resolution of the wave equation based on the finite-difference method.[13] These results validate the new approach for analysing the Doppler effect.

2. CONCERNING THE VANISHING OF THE DE

It is clear that for a wave source at rest (v = 0) there is no DE, i. e., there is no change between frequency of the transmitted signal and the frequency of the received signal. Let us consider the DE when the wave source is moving in an arbitrary direction and the observer is at rest, as illustrated in Figure 1. Is there an angle for which the DE vanishes? Yes, there is! There is zero DE if , and we can find the condition that fulfils this request for the three theories of the DE.

2.1 Zero DE for the classical DE

Within the classical DE theory, [Equation (2)], the DE "vanishes" for because,

$$\lambda_o = \lambda_s \Leftrightarrow c = c + v cos\vartheta \tag{5}$$

From this, we have for angle

$$\vartheta = acos(0). \tag{6}$$

As it can be seen, the null DE does not depend on the velocity of the source but only on the angle. In textbooks it is stated the assertion that according to the classical DE, the DE vanishes for , without physical explanation.[14]

2.2 Zero DE for the relativistic DE

For the relativistic DE formula, according to equation (3), one can found that the DE should vanish for:

$$\lambda_o = \lambda_s \Leftrightarrow \sqrt{c^2 - v^2} = c + v cos\vartheta \tag{7}$$

After simplification, we obtain the following zero DE condition within the relativist DE theory.

$$\vartheta = acos\left(\frac{\sqrt{c^2 - v^2} - c}{v}\right). \tag{8}$$

2.3 Zero DE for the new theory of DE

According to equation (4) for vanishing of the DE we have:

$$\lambda_o = \lambda_s \Leftrightarrow \sqrt{c^2 - v^2 sin^2\vartheta} + v cos\vartheta = c. \tag{9}$$

After some algebra, we find that the position angle of the observer for which the DE vanishes is given by:

$$\vartheta = acos\left(\frac{v}{2c}\right). \tag{10}$$

As it can be seen, unlike the case for classical DE, for the other two theories the achievement of zero DE depends on the position of the observer (angle) and on the velocity of the source. Then, for a given velocity, we have a certain position where zero DE is reached and vice versa, at a certain position, the source must have a certain velocity for zero DE to occur. For example, let us fix the position where the observer rests, at point P (Figure 1). We consider that distance OP (see Figure 1) is $n\lambda_s$. For the source at rest, since the source, observer distance, and the wave velocity are known, then the time the observer receives the certain wavefront *n* is also known. To have zero DE in the case of moving source, within the time for which the wavefront met with the observer, the source must reach a position in which $n\lambda_o = n\lambda_s$, so the source had to have a certain velocity. On contrary, if the source has another speed, then the possibility of zero DE is lost and the observer must change position to adjust to the source velocity to reach zero DE, which would eventually be reached for another wavefront. Thus, from physical point of view, it is logical that the null DE will depends on the position of the observer and on the velocity of the source.

3. COMPARISON OF THE RESULTS WITH THE DIFFERENT APPROACHES

Figure 2 shows the angle for zero DE againts normalised velocity v/c, for the different theories. These curves were obtained by using Equation (6), Equation (8) and Equation (10). From this figure, it can be seen that for low velocity of the source, the three theories gave an angle close to 90°. At higher velocity, the zero DE is obtained for approaching source in the new theory and for receding source in the relativistic DE theory. At very high velocity, close to the speed of light *c*, the zero DE is obtained at 60° within the new theory and at 180° with the relativistic DE theory.

Another distinction can be observed for the new theory of DE on one side and, classical and relativistic DE on the other side. As we pointed above, the new theory of the DE is based on the relative velocity between the wavefront and its source,

$$u_{cv} = \sqrt{c^2 - v^2 sin^2 \vartheta} + vcos\vartheta \quad (11)$$

which represents the velocity addition formula between the velocity of wave (c) and the velocity of its source (v). As we have seen above, there are situations where the DE vanishes, although v is non-zero. It means that for a given, it does exist a certain position of observer in which effect of is eliminated. In other words, it means that relative velocity (u_{cv}) between wavefronts and source must be equal to c ($u_{cv} = c$).

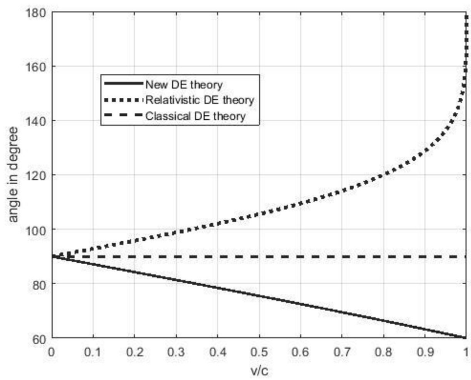

Figure 2: Angle for zero DE within the different theories.

If we compare Equation (5), Equation (7) and Equation (9), we find that relative velocity for the classical DE should be:

$$u_{cv} = c + vcos\vartheta \quad (12)$$

and for the relativistic DE, it must be written as:

$$u_{cv} = \sqrt{c^2 - v^2} - vcos\vartheta. \quad (13)$$

The main difference between Equation (11), Equation (12) and Equation (13) is that – for the first equation, there is an analytical method for its derivation (by using the law of cosines), while for the two latter there is no such method.[11,12,15] The eventual usage limit for each approach to the DE formula [Equation (2), Equation (3) and Equation (4)] is dependent to the way of insertion of the into the DE formula. As it can be seen the new approach has incorporated analytically both components of velocity v, $sin\vartheta$, and $cos\vartheta$, while two others only the component $cos\vartheta$.[12]

4. CONCLUSIONS

We have analysed the condition for zero DE when the source is moving, within three approaches: the classical DE, the relativistic DE, and alternative approach. Both the relativistic and the alternative approaches show that the angle for the null DE depends on the velocity of the source, by opposition with the classical DE theory, where this angle is always equal to 90°. The relativistic DE theory and the alternative approach presented different results in terms of the angle for null DE againts v/c. In the relativistic DE, this angle corresponds to a receding source whereas it corresponds to an approaching source in the alternative approach. It is clear that if the alternative approach is used in cosmology, we may obtain new interpretations for the observed Doppler shifts.

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