Journal of Physical Science, Vol. 35(1), 1–12, 2024

Transient Heat Transfer Analysis in Metal Plates with Variable Thickness

Mohammed Taibi,[1*] Younes Abouelhanoune[2*] and Fouad Dimane[1]

[1]LSIA, ENSAH Al-Hoceima, Abdelmalek Essaadi University, 93000 Tetouan, Morocco
[2]LSA, EMAO, ENSAH Al-Hoceima, Abdelmalek Essaadi University, 93000 Tetouan, Morocco

[*]Corresponding author: yabouelhanoune@uae.ac.ma; mohamed.taibi12@gmail.com

Published online: 30 April 2024

To cite this article: Taibi, M., Abouelhanoune, Y. & Dimane, F. (2024). Transient heat transfer analysis in metal plates with variable thickness. *J. Phys. Sci.*, 35(1), 1–12. https://doi.org/10.21315/jps2023.35.1.1

To link to this article: https://doi.org/10.21315/jps2023.35.1.1

ABSTRACT: *Nonlinear transient heat transfer via conduction–radiation is a dynamic topic of long-standing interest with applications ranging from aeronautical and mechanical engineering to industrial and civil security. To gain a better understanding of the performances of materials having thermal proprieties that change during nonlinear heat transfer, several studies using the finite element method (FEM) have been conducted. Such studies apply nonlinear thermal material characteristics to describe the complete system under different loading conditions in each region by adjusting the temperature values for the other three edges and the thickness parameter with Dirichlet boundary conditions. As a result, while modeling and simulating temperature distributions for such situations, nonlinearities generated by temperature-dependent thermal conductivity must be considered. In this work, we focus on the analysis of coupled transient heat transfer through two metal plates with temperature-dependent thermal characteristics in which the temperature is fixed along the bottom edge and heat is transferred from both the top and bottom faces of the two plates. FEM is employed to solve the nonlinear heat equation and compute the temperature as a function of time for variable thickness. The study examines the effect of modifying the thickness parameter values on the temperature distribution over time for various edge values over 5,000 s.*

Keywords: heat transfer, conduction-radiation, thermal plate, thickness parameter, FEM method

1. INTRODUCTION

Heat transfer analysis in metals is applied in many critical scientific fields, including the heat-treatment of materials, radiation heat waves and mechanical engineering, and it is used to assess the temperature distribution in conductive materials when the thermophysical properties, boundary conditions and/or intensity of the heat source are known.[1–7] The physical mechanism of conduction–radiation coupling is crucial and intricate.[8,9] To explain the radiation process, we apply the Stefan constant, emissivity coefficient and ambient temperature, assuming that heat transfer is proportional to the fourth order of the thermal gradient between the plane and the temperature difference.[10–12]

When material non-linearity is considered, the computational cost and effort for the analysis of transient heat transfer are enormously increased. Indeed, iteration methods must be used to solve this issue for convergence to be attained at each time step. Previous approaches related to this topic encompass Finite Element Method (FEM), Boundary Element Method (BEM), Meshless Method and various others are initially developed for engineering study to model and analyse systems complex in the fields of mechanical, civil and aeronautical engineering.[13–16] FEM is regarded as the most practical and powerful computational technique in the engineering sciences.[17–19] Nonlinear thermal material properties such as temperature-dependent thermal conductivity and specific heat capacity, as well as nonlinear thermal boundary conditions such as heat convection and radiation are used in FEM to account for the nonlinear characteristics of transient heat transfer.[18] The approach divides the complex domain into finite fragments whose behaviour can be described with relatively linear equations that are solved simultaneously for the entire system.[20,21] These numerical approaches are adaptable and effective for dealing with coupled transient heat transport problems.

In this study, we analysed coupled transient heat transfer through two square metal plates, one of copper and the other of aluminum, with temperature-dependent thermal characteristics, where the temperature is fixed along the bottom edge and heat is transferred from both top and bottom faces. The FEM approach is used to solve the nonlinear heat equation coupled with radiation considered as the source term via a square metal plate domain. The temperature variation with time is calculated by adjusting the temperature values for the other three edges of the plate with change in thickness parameter.

The plate used in this research has a consistent specific heat and density and is completely insulated from the other edges. Because the thickness of the plate is very low compared to the other planar dimensions, the temperature in the thickness direction may be considered constant. This suggested approach may be applied to a wide range of thermal material qualities and thicknesses, as well as nonlinear features in transient heat transfer situations. The purpose of this study is to investigate and evaluate the effects of varying the thickness parameter values for the copper and aluminum plate on the temperature distribution over time for various edge values.

2. THERMAL GOVERNING EQUATIONS

Assuming that the three edges are insulated, the heat is transferred from the faces of the plate by radiation through fixing the temperature along the bottom edge. Under these conditions, the two-dimensional transient nonlinear heat transfer is expressed as:

$$\rho c_p t_h \frac{\partial T}{\partial t} = \kappa(T) t_h \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \sigma \varepsilon \left(T^4 - T_0^4 \right) \tag{1}$$

where T denotes the temperature, ρ is the material density, c_p represents the specific heat, σ is the Stefan-Boltzmann constant, ε is the face body emissivity, T_0 is the ambient temperature, t_h is the plate thickness, and κ(T) denotes the thermal conductivities in different directions.

In this study, we assume that the nonlinear thermal conductivity is assumed to vary linearly with temperature. Its variation with temperature is given by κ(T) = k_0 (1+βT), where k_0 represents the thermal conductivity of the plate. Moreover, we replace the nonlinear quantity T^4 using the Taylor series expansion with the following linear approximation $T^4 = 4T_0^4 - 3T_0^4$, by considering that the limit will be small and increase at powers higher than 1 and can be neglected.[9]

The model describing the transient temperature in this metal plate is expressed as:

$$\frac{\partial T}{\partial t} = \frac{k_0}{\rho c_p}(1 + \beta T)\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{\sigma \varepsilon}{\rho c_p th}\left(4T_0^4 T - 4T_0^4 \right) \tag{2}$$

assuming $\alpha = \dfrac{k_0}{\rho c_p}$; $\gamma = \dfrac{\sigma \varepsilon}{\rho c_p}$ and $\delta = \dfrac{\gamma}{th}$, we get

$$\frac{\partial T}{\partial t} = \alpha(1 + \beta T)\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \delta 4T_0^4 (T - 1) \tag{3}$$

3. FEM APPROACH

3.1 Transient Nonlinear Thermal Analysis

The thermal properties of most materials vary with temperature. As a result, the governing equations and boundary conditions that characterise the temperature distribution in these materials become nonlinear. However, because addressing these nonlinear heat transport problems is challenging, simplifying assumptions are typically applied to linearise such situations. Mathematically, it can be confirmed that with an increasing number of elements, the FEM approach converges towards the exact solution. Therefore, any accuracy level required can be accomplished.[20] The temperature or heat limits conditions can be defined at any point within the finite element system and can take the form of pressure functions. The area of the solution is typically not straightforward to determine, and it can change size and form in unpredictable ways. When nonlinearity occurs, the geometry and material characteristics become important for convergence of the solution by the FEM approach.

3.2 Physical Properties and Boundary Conditions

Our goal was to determine the effects of plate thickness on nonlinear thermal conductivity coupled with radiation on heat transfer. This is done by fixing the nonlinear conduction parameter $\beta = 1$ and varying the thickness parameter by applying the properties indicated in Table 1 on a metal comprising copper and aluminum.

Table 1: The physical properties for metals plate.

Metals	Conductivity (W/m.K)	Density (kg/m^3)	Specific heat (J/Kg.K)	Emissivity	Stefan Boltzman Cte
Copper	400	8,920	385	0.03	5.6703 10^{-8} (W/m^2.K^4)
Aluminum	237	2,700	903	0.05	

Applying boundary conditions involves setting the temperature at the bottom edge to 100°C, while the other three edges are maintained at ambient temperature. Figure 1 shows the diagram of the triangular mesh in the plate with about 20 elements in each direction. The triangular grid iteration technique is very easy to implement and has been used successfully for different types of engineering problems.

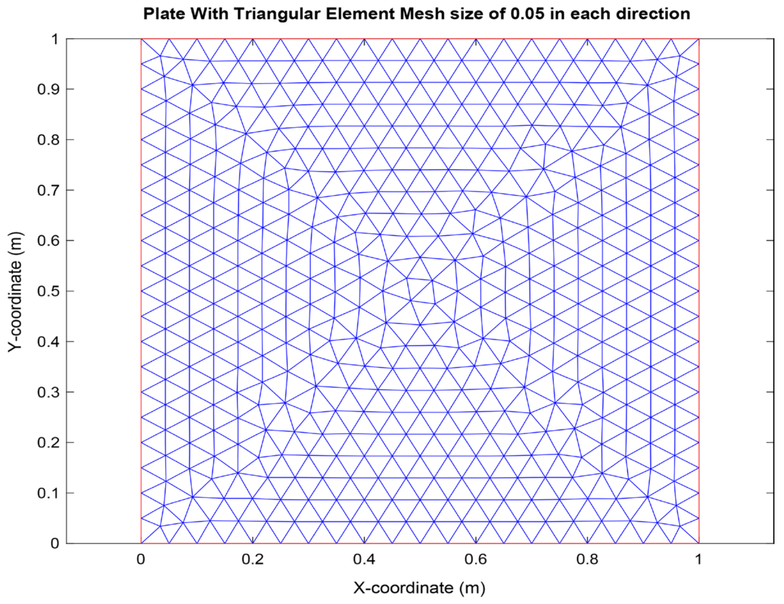
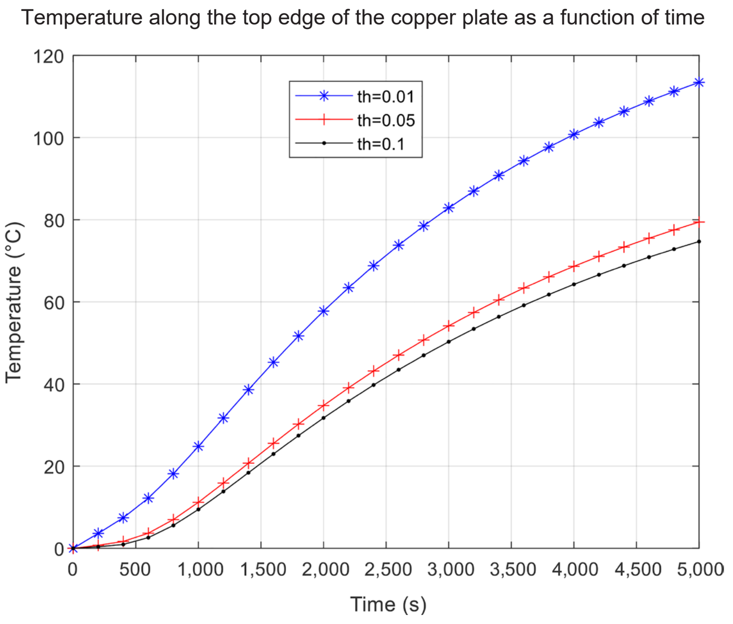


Figure 1: Plate with triangular finite element mesh.

4. RESULTS AND DISCUSSION

We evaluated the effect of changing the thickness parameter values on the evolution of temperature over time for the different edges of both plates over 5,000 s and obtained three temperature curves as a function of time for the top, E2 and E4 edges for both metals.

According to the profile of the evolution of the transient temperature in Figures 2a and 2b for the two metals, it is clear that for an extremely thin plate, the increase in thickness has a visible influence on the change in temperature at the centre of the edge of the plate which follows exponential growth.



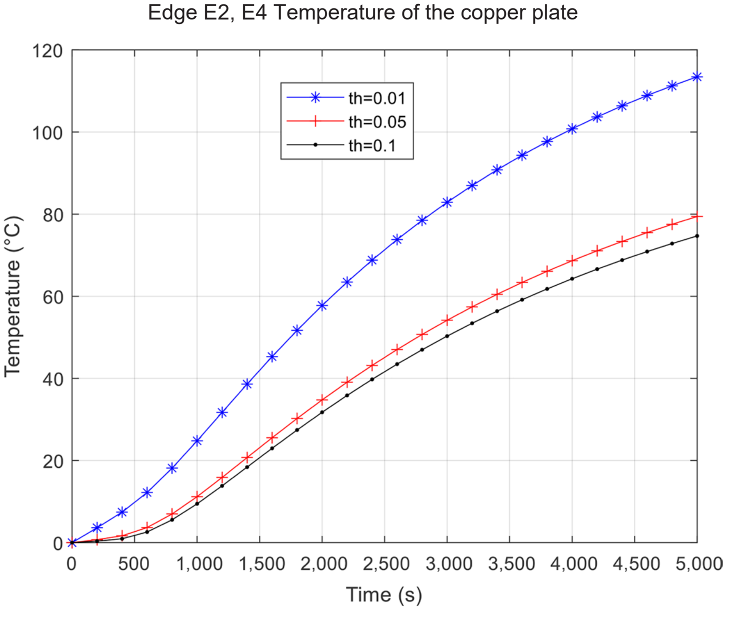
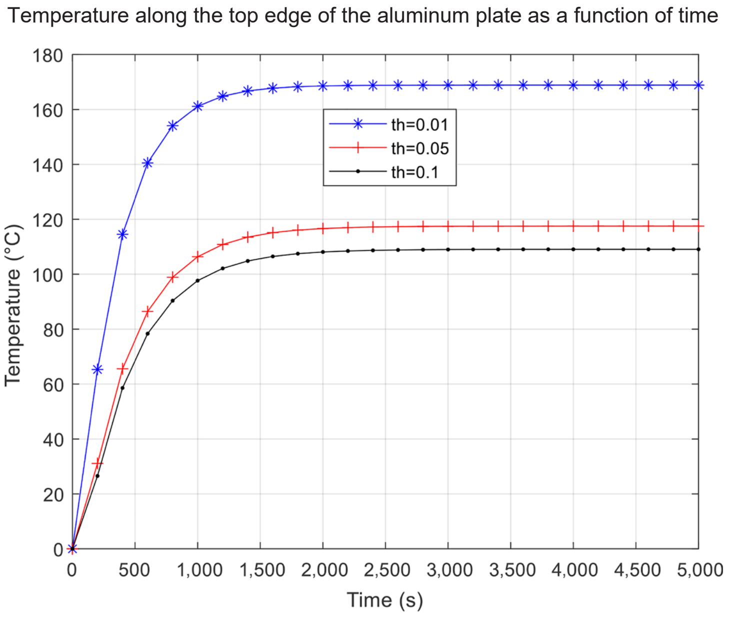


Figure 2a: Temperature along the edges of the copper plate in 5,000 s for $\beta = 1$.



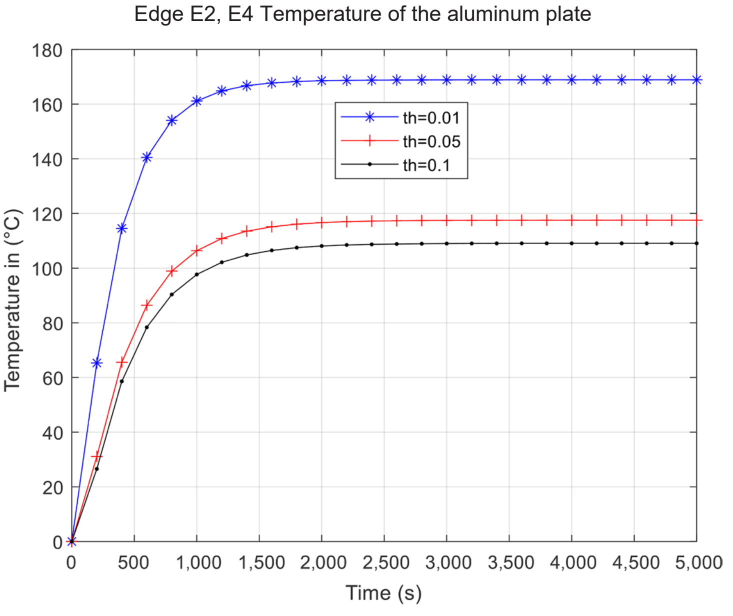


Figure 2b: Temperature along the edges of the aluminum plate in 5,000 s for $\beta = 1$.

In Figures 3a and 3b, we define the initial temperature to be $T_0 = 25°C$ everywhere except the bottom edge and define the time domain as 5,000 s to find the transient solution using FEM by changing the temperature values for the other three edges. We set the physics and analysis type, generated the finite element mesh in the model and set the time steps and initial conditions including convergence criteria and iteration termination conditions, among others.

It should be noted that the temperatures rise from the initial state after 5,000 s; this simulation time was chosen since it is sufficient to detect the high temperatures.

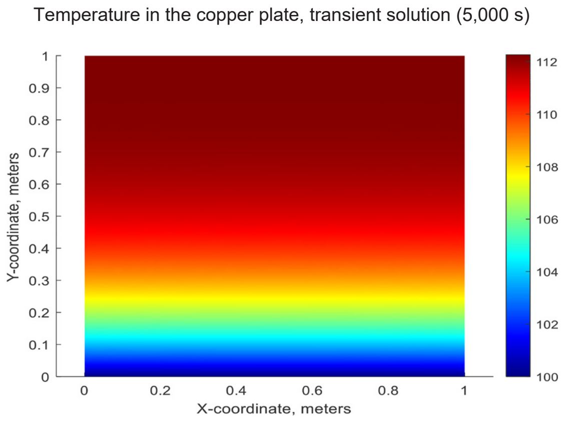


Figure 3a: Transient temperature in the copper plate.

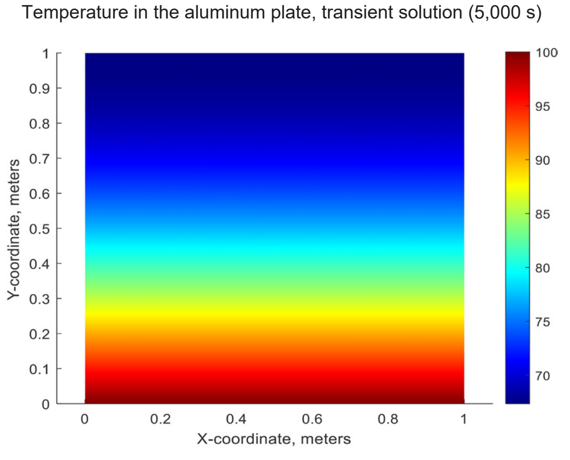


Figure 3b: Transient temperature in the aluminum plate.

In Table 2, the temperature variation at the top edge is significant for both metals depending on the thickness value.

Table 2: Value of temperature along the top edge plate in 5,000 s.

Thickness parameters	$t_h = 0.01$	$t_h = 0.05$	$t_h = 0.1$
Temperature at the top edge for copper (°C)	148.5	86.9	78.3
Temperature at the top edge for aluminum (°C)	155.2	113	106.9

The thickness of the plate and the physical properties of the metal play a very important role in the transfer of heat within the plate.

Figures 4a and 4b show numerically the variations in transient solutions in the plate using the same ambient temperature throughout all edges for both plates. We can observe a small difference of temperature value in the centre of plate. This fact shows that the two solutions are very close.

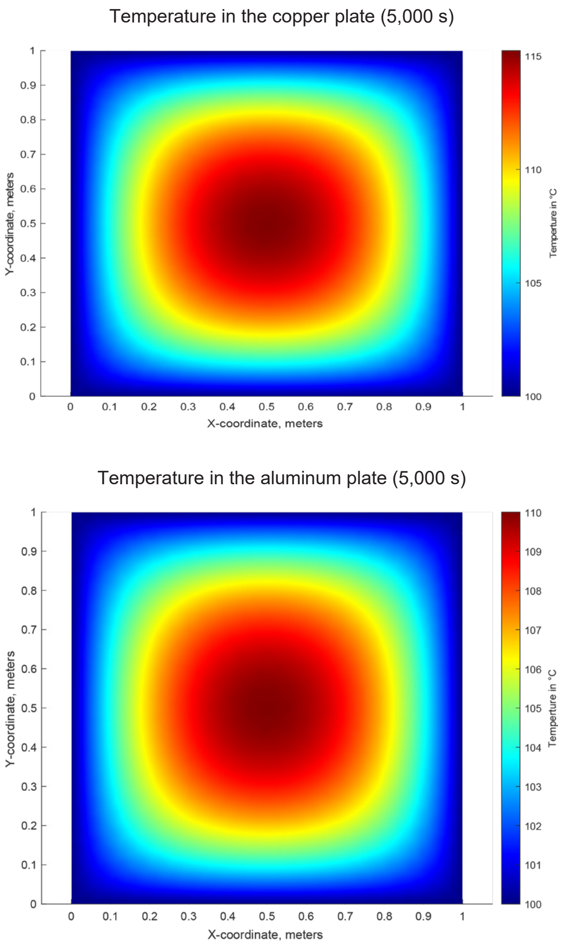


Figure 4a: Transient temperature for both plates with same temperature over all edges.

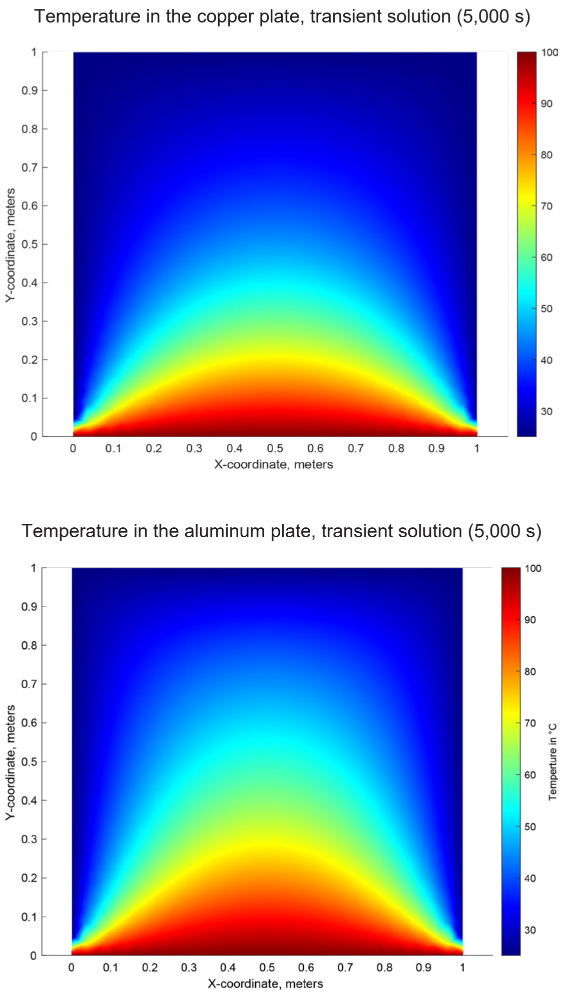


Figure 4b: Transient temperature for both plates with ambient temperature over all edges.

5. CONCLUSION

In this article, we used the FEM approach to examine the transient temperature in two plates as a function of time to evaluate the impact of thickness of each metal plate. As shown, the temperature profiles for the plate from the transient solution at the end are extremely similar for the plates, and the temperature is set at the bottom edge with no heat transfer from the other three edges. The transient solution attained steady-state values for both plates after roughly 5,000 s. Thus, material properties such as conductivity and thickness play very important roles in the heat transfer behaviour within the plate.

6. NOMENCLATURE

$\kappa(T)$	The temperature thermal conductivity	T	Local mean temperature [°C]
L	Length of heat plate [m]	Cp	Specific heat at constant pressure [J/Kg.K]
k0	Reference thermal conductivity [W/mK]	α	Design thermal diffusivity [m^2/s]
β	Nonlinear conductivity parameter	ρ	Temperature density dependent [kg/m^3]
ϵ	Face body emissivity		

7. REFERENCES

1. Dinarvand, S., Yousefi, M. & Chamkha, A. J. (2022). Numerical simulation of unsteady flow toward a stretching/shrinking sheet in porous medium filled with a hybrid nanofluid. *J. Appl. Comput. Mech*., 8(1), 11–20. https://doi.org/10.22055/jacm.2019.29407.1595
2. Delfour, M. C., Payre, G. & Zolesio, J.-P. (1987). Approximation of nonlinear problems associated with radiating bodies in space. *SIAM J. Numer. Anal*., 24(5), 1077–1094. https://doi.org/10.1137/0724071
3. Modest, M. F. (1993). *Radiative heat transfer*. New York: McGraw-Hill.
4. Siegel, R. & Howell, J. H. (1972). *Thermal radiation heat transfer*. New York: McGraw-Hill.
5. Aviles-Ramos, C., Haji-Sheikh, A. & Beck, J. V. (1998). Exact solution of heat conduction in composite materials and application to inverse problems. *J. Heat Trans*., 120(3), 592–599. https://doi.org/10.1115/1.2824316
6. Myers, G. E. (1971). *Analytical methods in conduction heat transfer*. New York: McGraw-Hill.
7. Beck, J. V. et al. (2008). Conduction in rectangular plates with boundary temperatures specified. *Int. J. Heat Mass Trans*., 51(19), 4676–4690. https://doi.org/10.1016/j.ijheatmasstransfer.2008.02.020
8. Feng, S. Z. et al. (2020). Transient nonlinear heat transfer analysis using a generic grid refinement for structure parameter variations. *Int. J. Therm. Sci.*, 153, 106357. https://doi.org/10.1016/j.ijthermalsci.2020.106357
9. Benleghrib, S., Abouelhanoune, Y. & El Jarroudi, M. (2018). Nonlinear heat transfer in rectangular plate with radiation effect. *JP J. Heat Mass Trans.*, 15, 483–496. https://doi.org/10.17654/HM015030483
10. Poirier, D. R. & Geiger, G. H. (2016). Conduction of heat in solids. In D. R. Poirier & G. H. Geiger (Eds.). *Transport phenomena in materials processing*. Cham: Springer, 281–327. https://doi.org/10.1007/978-3-319-48090-9_9
11. Verma, S. & Kulkarni V. S. (2014). Thermal stress analysis of rectangular plate due to convection using finite element method. *Int. J. Eng. Innov. Technol*., 3(7), 111–117.
12. Singh, I. V. (2004). A numerical solution of composite heat transfer problems using meshless method. *Int. J. Heat Mass Trans.*, 47, 2123–2138. https://doi.org/10.1016/j.ijheatmasstransfer.2003.12.013

13. Lo, C.-Y. (2011). A study of two-step heat conduction in laser heating using the hybrid differential transform method. *Num. Heat Trans., Part B: Fundamentals*, 59(2), 130–146. https://doi.org/10.1080/10407790.2011.550524
14. Mishra, T. N. (2015). Analytical solution of 2D SPL heat conduction model. *Int. J. Math. Trends Technol.*, 25(1), 47–54. https://doi.org/10.14445/22315373/IJMTT-V25P507
15. Crittenden, P. E. & Cole, K. D. (2002). Fast-converging steady-state heat conduction in a rectangular parallelepiped. *Int. J. Heat Mass Trans.*, 45(17), 3585–3596. https://doi.org/10.1016/S0017-9310(02)00066-2
16. Taibi, M., Abouelhanoune, Y. & Dimane, F. (2022). Numerical analysis of nonlinear heat conduction in a 2d thin metal plate. *JP J. Heat Mass Trans.*, 29, 189–200. https://doi.org/10.17654/0973576322051
17. Jordan, P. M. (2003). A nonstandard finite difference scheme for nonlinear heat transfer in a thin finite rod. *J. Diff. Eq. App.*, 9(11), 1015–1021. https://doi.org/10.1080/1023619031000146922
18. Feng, S. Z., Cui, X. Y. & Li, A. M. (2016). Fast and efficient analysis of transient nonlinear heat conduction problems using combined approximations (CA) method. *Int. J. Heat Mass Trans.*, 97, 638–644. https://doi.org/10.1016/j.ijheatmasstransfer.2016.02.061
19. Ascher, U. M., Mattjeij, R. M. M. & Russel, R. D. (1995). *Numerical solution of boundary value problems for ordinary differential equations*. Philadelphia: Society for Industrial and Applied Mathematics..
20. Sobamowo, M. G., Oguntala, G. A. & Yinusa, A. A. (2019). Nonlinear transient thermal modeling and analysis of a convective-radiative fin with functionally graded material in a magnetic environment. *Model. Simul. Eng.* https://doi.org/10.1155/2019/7878564
21. Huang, G., Wang, H. & Li, G. (2017). A novel multi-grid assisted reanalysis for re-meshed finite element models. *Comput. Methods Appl. Mech. Eng.*, 313, 817–833. https://doi.org/10.1016/j.cma.2016.10.029